BANGABASI COLLEGE

B.Sc. First Year (Part-I) Honours Test Examination-2016

Subject – Mathematics

Paper - I

Full Marks – 100

Time – 4 Hours

1. Answer any two questions: 2× 5

(a) Prove that the orthocentre (p,q) of the triangle formed by the straight lines $ax^2 + 2hxy + by^2 = 0$ and $lx + my = 1$ is given by
$\frac{p}{l} = \frac{q}{m} = \frac{a+b}{am^2-2hlm+bl^2}$.

(b) If A and B be two points on the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ such that three times the eccentric angle of the one is equal to the supplement of that of the other , then find the locus of the pole of AB with respect to the ellipse.

(c) Reduce the following equation to it's canonical form and determine the nature of the conic represented by it:
$4x^2 + 4xy + y^2 - 12x - 6y + 5 = 0$.

(d) If the normal be drawn at one extremity $(l , \frac{\pi}{2})$ of the latus rectum LSL' of the conic $\frac{l}{r} = 1+e\cos\theta$ where S is the pole , then show that the distance from the focus S of the other point in which the normal meets the conic is $\frac{l(1+3e^2+e^4)}{1+e^2-e^4}$.

2. Answer any four questions : 4× 5

(a) Show that there exists a unique positive real number x such that $x^2 = 2$.

(b) Prove that every bounded infinite subset of $\mathcal{R}$ has at least one limit point in $\mathcal{R}$. Is The result true for a unbounded infinite subset of $\mathcal{R}$? Justify your answer .

(c) Prove that the union of an enumerable number of enumerable sets is enumerable. Hence show that the set Q is enumerable .

(d) Prove that the sequence $\{u_n\}$ defined by $u_1 = \sqrt{6}$ and $u_{n+1} = \sqrt{6 + u_n}$ for n≥ 1, converges to 3 .

(e) Prove that every bounded sequence of real numbers has a convergent subsequence . Is the result true for a bounded below sequence of real numbers ? Justify your answer .

(f) Let [a,b] be a closed and bounded interval and a function f : [a,b]→$\mathcal{R}$ be continuous on [a,b] . If f(a) ≠ f(b) then prove that f attains every value between f(a) and f(b) at least once in the open interval (a,b) .Is the converse true

(g) Let f : (a,b)→$\mathcal{R}$ be monotone on (a,b) . Then prove that f can not have a discontinuity of the second kind in (a,b) .

3 . Answer any six questions: 5x6=30

[a] A variable plane passes through a fixed point (α,β,γ) and meets the coordinate axes at P, Q and R. Show that the locus of the point common to the planes through P,Q,R and arallel to the coordinate planes is $\frac{\alpha}{x}+\frac{\beta}{y}+\frac{\gamma}{z}=1.$

[b] Show that the surface generated by a straight line which intersects the lines $y=0,\ z=c; x=0, z=-c$ and curve $z=0,\ xy+c^2=0$ is $z^2-c^2=xy.$

[c] Find the position vector of the point of intersection of the line joining the points (1,-2,1) and (0,-2,3) with the plane through the points (0,0,0), (2,4,1) and (4,0,2).(By vector method)

[d] Show that if, $\vec{a}=a_1\vec{l}+a_2\vec{m}+a_3\vec{n},\quad \vec{b}=b_1\vec{l}+b_2\vec{m}+b_3\vec{n},\quad \vec{c}=c_1\vec{l}+c_2\vec{m}+c_3\vec{n}$ where $\vec{l},\vec{m},\vec{n}$ are three non-coplanar vectors, then $\left[\vec{a}\,\vec{b}\,\vec{c}\right]=\begin{vmatrix} a_1 & a_2 & a_3\\ b_1 & b_2 & b_3\\ c_1 & c_2 & c_3\end{vmatrix}\left[\vec{l}\,\vec{m}\,\vec{n}\right]$, where symbols have their usual meanings. You can assume that in a scalar triple product dot and cross can be interchanged.

[e] Show that a necessary and sufficient condition that a proper vector α always remains parallel to a fixed line is that $\alpha\times\frac{d\alpha}{dt}=0.$

[f] Prove that grad$\left(\vec{A}\cdot\vec{B}\right)=\left(\vec{B}\cdot\nabla\right)\vec{A}+\left(\vec{A}\cdot\nabla\right)\vec{B}+\vec{B}\times curl\,\vec{A}+\vec{A}\times curl\,\vec{B}.$

[g]Define injective mapping. A mapping f:NxN→N is defined by f(m,n)=2^m3^n .Show that 'f' is injective but not surjective. If A and B be two sets having n distinct elements, show that the number of bijective mappings from A to B is n!. 1+2+2

[h] (i)Show that the set of all complex numbers a+ib (where $i^2=-1$)for $a^2+b^2=1$ is a group under the multiplication of complex numbers.

(ii) If b be an element of a group and order of b is 20 find the orderof b^{15} 3+2

4. Answer question no (a) , and any three from rest. $4 + 3 \times 4 = 16$

(a) State and proof De Moivre's theorem.

(b) Show that the sum of the squares of all the values of $(\sqrt{3}+i)^{3/7}$ is 0.

(c) If x, y, z are positive real numbers and $x^2 + y^2 + z^2 = 12$ prove that the minimum value of $x^3 + y^3 + z^3$ is 24 and the maximum value of $x + y + z$ is 6.

(d) i) If a is prime to b and c is a divisor of a, prove that c is prime to b.

ii) If a is prime to b, prove that $a^2 + b^2$ is prime to a^2b^2.

(e) If α is an imaginary root of $x^{11} - 1 = 0$, prove that

(i) $(\alpha+1)(\alpha^2+1)\ldots\ldots\ldots(\alpha^{10}+1) = 1$

(ii) $(\alpha+2)(\alpha^2+2)\ldots\ldots(\alpha^{10}+2) = \frac{2^{11}+1}{3}$

5. Answer any one of the following questions:- 4X1=4

(a) If m,n are positive integers and m>n, show that $\int_0^{\pi} \cos^n x \cos mx \, dx = 0.$

(b) Prove that $\int \frac{dx}{(5+4\cos x)^2} = \frac{1}{27}(5z - 4\sin z)$, Where $\tan\frac{z}{2} = \frac{1}{3}\tan\frac{x}{2}$.

6. Answer any four questions: 4x5=20

(a) Prove that $\begin{vmatrix} a^2 & (s-a)^2 & (s-a)^2 \\ (s-b)^2 & b^2 & (s-b)^2 \\ (s-c)^2 & (s-c)^2 & c^2 \end{vmatrix} = 2s^3(s-a)(s-b0(s-c)$

where 2s=a+b+c.

OR

Let C[a,b] be the set of all real valued continuous functions on [a,b]. let $f(x) + g(x) = (f+g)(x)$ and $\alpha f(x) = (\alpha f)(x)$, for $f, g \in c[a,b]$ and $\alpha \in IR$. Show that C[a,b] is a vector space over IR.

(b Prove that every linearly independent subset of a finitely generated vector space V(F) is either a basis or can be extended to form a basis of V.

(c) Investigate for what values of λ and μ, the following equations

$$x + y + z = 6$$
$$x + 2y + 3z = 10$$
$$x + 2y + \lambda z = \mu$$

has (i) no solution; (ii) unique solution; (iii) an infinite number of solutions.

(d) Let $S = \{(x,y,z) \in IR^3 : x + 2y - z = 0, 2x + 3z - y = 0\}$. Prove that S is a subspace of the real vector space IR^3. Also find a basis of S and the dimension of S.

(e) Prove that the eigen values of a real skew-symmetric matrix are purely imaginary or zero.

(f) Reduce the quadratic form $Q = xy + yz + zx$ to its canonical form and hence find its rank and signature.

(g) Use Gram-Schmidt process to obtain an orthonormal basis of the subspace of the Euclidean space IR^3 with standard inner product, generated by the linearly independent set $\{(1,1,1), (2,-2,1), (3,1,2)\}$.

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