BANGABASI COLLEGE

B.Sc. First year Honours Mid-Term Test Examination-2014

Subject – Mathematics

Full Marks – 50

Time – 2Hours

1. Answer any two questions. 2x5=10

(a) If $A = \begin{pmatrix} 1 & 0 \\ -1 & 1 \end{pmatrix}$ show that $A^2 - A + 2I = 0$. Hence find A^{50}.

(b) (i) Define an idempotent matrix. If A be an idempotent matrix of order 'n', show that $I_n - A$ is also idempotent.

(ii) If AB = B and BA = A show that A and B are both idempotent. 3+2

(c) Prove without expanding that

$$\begin{vmatrix} 1 & m & m^2 & m^3 \\ m^3 & 1 & m & m^2 \\ m^2 & m^3 & 1 & m \\ m & m^2 & m^3 & 1 \end{vmatrix} = (1-m^4)^3.$$

(d) If S= M+iN be a skew Hermitian matrix, then prove that

(i) the diagonal elements of S are all imaginary or zero. 2

(ii) M is a real skew symmetric matrix and N is a real symmetric matrix. 3

2. Answer any two questions. 2x5=10

(a) If $a_1, a_2, a_3, \ldots\ldots a_n$ be n positive rational numbers and $s = a_1 + a_2 + \ldots\ldots + a_n$, prove that

$$\left(\frac{s}{a_1}-1\right)^{a_1}\left(\frac{s}{a_2}-1\right)^{a_2}\ldots\ldots\left(\frac{s}{a_n}\right)^{a_n} \le (n-1)^s.$$

(b) Show that the product of the values of $(1+\sqrt{3}i)^{\frac{3}{4}}$ is 8.

(c) Prove by vector method if two medians of triangle be equal then the triangle is isosceles.

3. Answer any two questions. 2x5=10

(a) (i) If a be an odd integer, prove that $24/a(a^2-1)$. 3

(ii) If gcd (a,b) = au + bv where u and v are integers, prove that gcd(u,v) = 1. 2

(b) (i) Use the theory of convergence to prove that $17/2^{3n+1}+3.5^{2n+1}$ for all integers $n \geq 1$. 3

(ii) If n be an even positive integer, prove that $\phi(2n) = 2\phi(n)$. 2

(c) If a,b,c are integers and a, b, are not both zero, then prove that the equation ax + by =c has an integral solution if and only if d is a divisor of c, where d = gcd(a,b). Hence find positive integral solutions of the equation 13x + 4y = 115. 5

(d) Prove that the orthocentre (α,β) of the triangle formed by the straight lines $ax^2+2hxy+by^2=0$ and $lx+my=1$ is given by

$$\frac{\alpha}{l}=\frac{\beta}{m}=\frac{a+b}{am^2-2hlm+bl^2}.$$ 5

4. Answer any two questions. 2x5=10

(a) State Archimedean property of IR. Use this property to show that for a positive real number x, there exists a natural number m such that $m-1 \leq x < m$.

(b) Let S and T be two non-empty bounded subsets of IR and $U = \{x+y : x \in S, y \in T\}$. Prove that sup U = sup S + sup T and inf U = inf S + inf T.

(c) Define derived set of a set in IR. If s = {1, 1/2, 1/3,..........1/n,..........}, find the derived set of S.

(d) When a sequence $\{x_n\}_n$ is said to be convergent ?
Prove that a convergent sequence is bounded. Is the converse true ? Justify your answer .

(e) Prove that the sequence $\{x_n\}$ defined by $x_1=\sqrt{7}$ and $x_{n+1}=\sqrt{7+x_n}$ for all $n \geq 1$, converges to positive root of the equation $x^2-x-7=0$.

5. Answer any two questions: 5x2=10

[a][i] Either prove or disprove with the help of a counter example $(A-B)' = (B-A)'$ 2

[ii] Consider the set S={1,2,3,4} and the partition { {1}, {2}, {3,4} } of S. Find the equivalence relation corresponding to the above partition of S. 3

[b] Define injective mapping. A mapping $f : N \times N \rightarrow N$ is defined by $f(m,n) = 2^m 3^n$. Show that 'f' is injective but not surjective. If A and B be two sets having n distinct elements, show that the number of bijective mapping from A to B is n!. 1+2+3

[c] Let Z_n be the set of integers modulo n, where n is a positive integer, i.e. $Z_n = \{[x] : x \in Z\}$. We define $+_n$ on Z_n by $[a] +_n [b] = [a+b], \quad \forall [a],[b] \in Z_n$. Show that $(Z_n, +_n)$ is a group. Is it commutative. 4+1

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