Full Marks: 100   **Time: 4 Hrs**

1. **Answer any** *ten*: $10 \times 2 = 20$
    a) What is Konigsberg bridge problem?
    b) What do you mean by a Complete graph?
    c) What are the quantifiers of a Predicate?
    d) What is Euler Graph?
    e) Define cut-set of a graph.
    f) What are Big-Omega notations?
    g) What is a minimal spanning tree?
    h) What do you mean by context free language?
    i) Define Moore Machine.
    j) What do you mean by NDFA?
    k) Differentiate between DFA and NDFA?
    l) Give an example of a regular grammar.
    m) Define complete binary tree.
    n) Prove that the time complexity of binary search is O(log n).

Answer any *eight* questions taking at least *one* from each group.

Group-A

2. a) Find minimum and maximum possible height of a binary tree with n vertices.
    b) Explain Depth-First-Search (DFS) algorithm with a non-trivial example. What modifications are to be done to get Breadth-First-Search (BFS) algorithm from it?
                                                           $4 + (4+2) = 10$
3. a) Prove that if the graph G does not have any isolated vertices, then it has all Euler cycle if and only if G is connected and the degree of each vertex is even.
    b) Prove that a simple graph with n vertices and k components can have at most $(n-k)(n-k+1)/2$ edges.
                                                           $6+4=10$
4. a) Define in-degree and out-degree of a vertex in a digraph. Give example.
    b) Describe Djikstra's Algorithm with suitable example.
                                                           $(2+2)+6=10$

Group-B

5. a) State and prove generalized pigeonhole principle.
    b) Find the number of subsets (with 10 elements) of the set S= {1, 2,......., 100} that do not contain a pair of consecutive integers.
    c) How can you relate big-Oh, big-Omega and big-Theta notations? Justify your answer.
                                                           $(2+2)+3+3=10$
6. a) Show that $(p \to q) \lor (q \to r)$ and $(p \lor q) \to r$ are logically equivalent.
    b) Show that $((p \lor q) \lor r) \to (p \lor (q \lor r))$ is a tautology.

    c) Give examples of relations that are:
        i) Only Reflexive
        ii) Only Transitive $3+3+ (2 \times 2) = 10$
7. a) Represent the primal L.P.P.
        Maximum $Z = c.\, x$, $c \in R^n$ subject to
            A. $x <= b$, $b^T \in R^m$, $x >= 0$ in its dual form.

    b) Obtain the dual problem of the following L.P.P.

        Max $f(x) = 2x_1 + 5x_2 + 6x_3$

            Subject to the constraints,

$$5x_1+6x_2-x_3<=3,$$
$$-2x_1+x_2+4x_3<=4,$$
$$x_1-5x_2+3x_3>=1,$$
$$-3x_1-3x_2+7x_3<=6,$$

And $x_1, x_2, x_3>=0$

c) State when the Newton's Forward formula cannot be used for computing the function f(x).

4+4+2=10

Group-C

8. Given the following NDFA: 3 + 2 + 5 = 10

State	Next State		
	Input=a	Input=b	Input=c
→q0	q1 q3	q3	q1 q2
q1	q0 q1	q1 q3	q0
q2	q1 q3	q0	q1 q2
(q3)		q2	-

a) Test whether the string *aabbcc* and *ccbbaa* are accepted by the transition system.
b) Construct transition diagram for the NDFA.
c) Construct a DFA equivalent to the NDFA.

9. a) Find the language equivalent to the grammar G= ({S,A}, {0,1}, {S→0S0|A, A→1A1|0}, S).
   b) Construct a grammar generating { $a^n b^m c^n$ | n>=1, m>=0}.
   c) What is a regular expression?   4 + 4 + 2 = 10

10. a) State Chomsky's Classification of grammars citing one example for each type.
    b) Give regular expressions of the following:
       i.   {00, 01, 10, 11}
       ii. All strings over the set {a,b} starting with aba and ending with bb.
       iii. All strings containing an even number of 1s.
       iv. All strings over {0, 1} containing exactly three 0s.

6 + 4 = 10

Group – D

11. a) The following list of array elements is given:-
       54, 33, 21, 75, 69, 50, 80, 29

    Assume that the array index starts from 1. Explain with the above list of elements how stacks are implemented to sort the array using quick sort technique (No formal algorithm is required). Can the concept of queue be implemented instead of stack in quick sort algorithm? Justify your answer.

    b) Define heap. Construct a max-heap with the following array of elements:-

       25, 10, 7, 16, 19, 23, 42, 33, 5, 11, 26

    Show with explanation, the different intermediate stages in the process of forming the heap.

(3+2) + (1+4) =10

12. a) Explain the concept of hashing technique. What are the criteria for choosing a good hash function?
    b) Explain collision in hashing. What are the broad classes of collision resolution techniques?
    c) Write a C-function to insert an element in the hash table using linear probing.

(2+2) + (2+2) + 2=10

13. a) Construct a binary search tree from the following traversals:
       preorder: a b d e i j c f h g
       In order: d b i e j a f h c j

    b) What is link list? How do we insert values in link list?

(5+(2+3))=10