PAIR OF STRAIGHT LINES

1. If the equation $ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0$ represents two intersecting straight lines, show that the square of the distance of the point of intersection of the straight lines to the origin is $\frac{c(a+b) - f^2 - g^2}{ab - h^2}$.

2. If the equation $ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0$ represents two straight lines equidistant from the origin, show that $f^4 - g^4 = c(bf^2 - ag^2)$.

3. If the pair of straight lines joining the origin to the points of intersection of the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ and the straight line $lx + my + n = 0$ are perpendicular to each other then show that $\frac{a^2 + b^2}{l^2 + m^2} = \frac{a^2b^2}{n^2}$.

4. If the equation $ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0$ represents two parallel straight lines, show that the distance between them is $2\sqrt{\frac{g^2 - ac}{a(a+b)}}$

5. If the equation $ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0$ represents a pair of straight lines, prove that the area of the triangle formed by their angle bisectors and the axis of x is $\frac{\sqrt{(a-b)^2 + 4h^2}}{2h} \cdot \frac{ca - g^2}{ab - h^2}$.

6. Show that if one of the lines given by the equation $ax^2 + 2hxy + by^2 = 0$ be perpendicular to one of the lines given by $a'x^2 + 2h'xy + b'y^2 = 0$ then $(aa' - bb')^2 + 4(ah' + hb')(ha' + bh') = 0$.

7. Show that the equation of the line joining the feet of the perpendiculars from the point $(d,0)$ on the lines $ax^2 + 2hxy + by^2 = 0$ is $(a-b)x + 2hy + bd = 0$.

8. If the equation $ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0$ represents a pair of straight lines ; show that the area of the parallelogram formed by them and the pair of parallel lines through the origin is $\frac{c}{2\sqrt{h^2 - ab}}$.

9. Show that the orthocenter of the triangle formed by the straight lines $ax^2+2hxy+by^2=0$ and $lx+my=1$ is a point (x_1,y_1) such that $\frac{x_1}{l}=\frac{y_1}{m}=\frac{a+b}{am^2+bl^2-2hlm}$.

10. Show that one of the bisectors of the angles between the pair of straight lines $ax^2+2hxy+by^2=0$ will pass through the point of intersection of the two straight lines $ax^2+2hxy+by^2+2gx+2fy+c=0$ if $h(g^2-f^2)=fg(a-b)$.

11. If the straight lines $ax^2+2hxy+by^2=0$ $(b\neq 0)$ be the two sides of a parallelogram and the straight line $lx+my=1$ be one of its diagonals, then show that the equation of the other diagonal is $y(bl-hm)=x(am-hl)$.

12. The straight lines joining the origin to the common points of intersection of the curve $ax^2+2hxy+by^2+gx+fy=c$ and the variable straight line $lx+my=1$ are at right angles. Find the locus of the foot of the perpendicular from the origin on the line $lx+my=1$.

13. If the straight lines $ax^2-2hxy+by^2=0$ from an equilateral triangle with the straight line $x\cos\alpha+y\sin\alpha=p$, then show that $\frac{a}{1-2\cos 2\alpha}=\frac{h}{2\sin 2\alpha}=\frac{b}{1+2\cos 2\alpha}$.

14. If the equation $ax^2+2hxy+by^2+2gx+2fy+c=0$ represents a pair of straight lines, then show that $ax^2+2hxy+by^2=0$ represents a pair of straight lines through the origin parallel to the first pair. Interpret the equation $2gx+2fy+c=0$ with justification.

GENERAL EQUATION OF SECOND DEGREE

1. Reduce the equation $4x^2 + 4xy + y^2 - 4x - 2y + a = 0$ to its canonical form and determine the nature of the conic for different values of "a"

2. Reduce the equation $x^2 - 5xy + y^2 + 8x - 20y + 15 = 0$ to its canonical form and show that it represents a hyperbola.

3. Reduce the equation $4x^2 - 4xy + y^2 - 8x - 6y + 5 = 0$ to its canonical form and show that it represents a parabola. Find the latus rectum and the equation of the axis of the parabola.

5. Reduce the equation $x^2 - 6xy + 9y^2 + 4xy - 12y + 4 = 0$ to the canonical form and determine the type of the conic represented by it.

6. Reduce the equation $x^2 - 2xy + y^2 + 6x - 14y + 29 = 0$ to its canonical form and determine the type of the conic represented by it.

7. Reduce the equation $6x^2 + 24xy - y^2 - 60x - 20y + 80 = 0$ to the standard form and hence find the eccentricity of the conic represented by it.

8. Reduce the equation $3(x^2 + y^2) + 2xy = 4\sqrt{2}(x + y)$ to its canonical form. Name the conic and determine equation of its axes.

9. Reducing the equation $x^2 + 2xy + y^2 - 4x + 8y - 6 = 0$ to its canonical form, show that it is a parabola and determine the coordinates of its vertex.

10. Reduce the equation $3x^2 - 8xy - 3y^2 + 10x + 13y + 8 = 0$ its canonical form and determine the type of the conic represented by it.

11. Reduce the equation $11x^2 + 4xy + 14y^2 - 26x - 32y + 23 = 0$ to standard form and hence find the eccentricity of the conic represented by it.

12. Reducing the equation $11x^2 - 4xy + 14y^2 - 58x - 44y + 71 = 0$ to its canonical form. Name the conic, find the eccentricity of the conic.

STRAIGHT LINE IN SPACE

1. Show that the surface generated by a straight line which intersects the lines $y = 0,\ z = c;\ x = 0, z = -c$ and curve $z = 0,\ xy + c^2 = 0$ is $z^2 - c^2 = xy$.

2. A variable plane has intercepts on the co-ordinate axes, the sum of whose squares is k^2 Show that the locus of the foot of the perpendicular from the origin to the plane is $\left(x^2 + y^2 + z^2\right)^2\left(\frac{1}{x^2} + \frac{1}{y^2} + \frac{1}{z^2}\right) = k^2$

3. A variable line intersects x -axis, the curve $x = y,\ y^2 = cz$ and is parallel to the plane YOZ. Prove that it generates the surface $xy = cz$.

4. A variable straight line, parallel to yz-plane intersects the curves $x^2 + y^2 = a^2,\ z = 0$ and $x^2 = az,\ y = 0$. Prove that it generates the surface $x^4y^2 = \left(a^3 - x^2\right)\left(x^2 - az\right)^2$

5. Show that the locus of a point which is equidistant from two given straight lines $y = mx, z = c$ and $y = -mx, z = -c$ is $mxy + c\left(1 + m^2\right)z = 0$.

6. Prove that the locus of the lines which intersects the three lines y-z = 1, x = 0; z-x =1,y = 0; x-y =1,z = 0 is $x^2 + y^2 + z^2 - 2yz - 2zx - 2xy = 1$.

7. If θ be the angle between two lines whose direction cosines are l_1, m_1, n_1 and l_2, m_2, n_2, show that the direction cosines of their angular bisectors are
$$\frac{l_1 + l_2}{2\cos\frac{\theta}{2}}, \frac{m_1 + m_2}{2\cos\frac{\theta}{2}}, \frac{n_1 + n_2}{2\cos\frac{\theta}{2}} \quad \text{and} \quad \frac{l_1 - l_2}{2\sin\frac{\theta}{2}}, \frac{m_1 - m_2}{2\sin\frac{\theta}{2}}, \frac{n_1 - n_2}{2\sin\frac{\theta}{2}}$$

8. Show that the equation to the plane containing the straight line $\frac{y}{b} + \frac{z}{c} = 1, x = 0$ and parallel to the straight line $\frac{x}{a} - \frac{z}{c} = 1, y = 0$ is $\frac{x}{a} - \frac{y}{b} - \frac{z}{c} + 1 = 0$ and if 2d be the shortest distance between the lines then show that $\frac{1}{d^2} = \frac{1}{a^2} + \frac{1}{b^2} + \frac{1}{c^2}$.

9. Show that the locus of a variable line which intersects the three lines $y = mx, z = c;\ y = -mx, z = -c;\ y = z, mx = -c$ is the surface $y^2 - m^2x^2 = z^2 - c^2$

10. Show that the equation to the plane through the line $\frac{x}{l} = \frac{y}{m} = \frac{z}{n}$ and perpendicular to the plane containing the lines $\frac{x}{m} = \frac{y}{n} = \frac{z}{l}$ *and* $\frac{x}{n} = \frac{y}{l} = \frac{z}{m}$ is $(m - n)x + (n - l)y + (l - m)z = 0$.

11. Find the equation of the perpendicular from (1,6,3) to the line $x+y-z+1=0=2x-7y+4z-1$. Also determine the foot of the perpendicular.

12. Find the length of the shortest distance between the lines $\frac{x-3}{2}=\frac{y-5}{2}=\frac{z+1}{1}$ and $y-z-1=0=x-2$. Find also the coordinates of the points where the line of shortest distance meets the given lines.

13. Prove that the line of shortest distance between the z-axis and the variable line $\frac{x}{a}+\frac{z}{c}=\lambda\left(1+\frac{y}{b}\right), \frac{x}{a}-\frac{z}{c}=\frac{1}{\lambda}\left(1-\frac{y}{b}\right)$ when λ varies, generates the surface $abz(x^2+y^2)=(a^2-b^2)cxy$.

14. Find the condition that the lines $\frac{x-\alpha}{l}=\frac{y-\beta}{m}=\frac{z-\gamma}{n}, \frac{x-\alpha'}{l'}=\frac{y-\beta'}{m'}=\frac{z-\gamma'}{n'}$ may be coplanar. If the condition is satisfied, find the point of intersection of the lines, given that they are not parallel.

15. A variable st. line intersects the lines $y=0, z=c; x=0, z=-c$ and is parallel to the plane $lx+my+nz=p$. Prove that the surface generated by it is $lx(z-c)+my(z+c)+n(z^2-c^2)=0$.

16. Find the shortest distance between the two lines $x+y+2z-1=0=x-2y-z-1$ *and* $2x-y+z-3=0=x+y+z-1$.

17. A variable straight line intersects the straight lines $x=q, y=-r; \quad y=r, z=-p; \quad z=p, x=-q$.
Show that the equation of the locus of the straight line is $pxy+qyz+rzx+pqr=0$

18. Find the shortest distance between the axis of z and the line $ax+by+cz+d=0=a'x+b'y+c'z+d'$

19. Find the magnitude and equation of the line of shortest distance between the lines $\frac{x}{4}=\frac{y+1}{3}=\frac{z-2}{2}$ and 5x-2y-3z +6 = 0 = x-3y+2z-3.

20. Find the surface generated by the straight line which intersects the lines $x+y=0, z=0$; $x-y=z, x+y=2a$; and the parabola $x^2=2az, y=0$

21. Two straight lines $\frac{x-\alpha_1}{l_1}=\frac{y-\beta_1}{m_1}=\frac{z-\gamma_1}{n_1}$ and $\frac{x-\alpha_2}{l_2}=\frac{y-\beta_2}{m_2}=\frac{z-\gamma_2}{n_2}$ are cut by a third line whose direction cosines are λ,μ,η Show that d, the length intercepted on the third line by the first two lines is given by $d\begin{vmatrix} l_1 & m_1 & n_1 \\ l_2 & m_2 & n_2 \\ \lambda & \mu & \eta \end{vmatrix}=\begin{vmatrix} \alpha_1-\alpha_2 & \beta_1-\beta_1 & \gamma_1-\gamma_2 \\ l_1 & m_1 & n_1 \\ l_2 & m_2 & n_2 \end{vmatrix}$

22. A variable straight line always intersects the following three lines $x=k, y=0; y=k, z=0; z=k, x=0.$ Prove that the equation to its locus is $xy+yz+zx-k(x+y+z-k)=0.$

23. Find the magnitude and the equation of the line of shortest distance between the lines $2x+y-z=0=x-y+2z$ and $x+2y-3z-4=0=2x-3y+4z-5.$

THE PLANE

1. A variable plane which is at a constant distance 3p from the origin cuts the axes in A,B,C Show that the locus of the point of intersection of the planes through A,B,C parallel to the coordinate planes is $9\left(\frac{1}{x^2}+\frac{1}{y^2}+\frac{1}{z^2}\right)=\frac{1}{p^2}$.

2. Perpendiculars PL, PM, PN are drawn from the point P (a,b,c) to the coordinate planes. Show that the equation of the plane LMN is $\frac{x}{a}+\frac{y}{b}+\frac{z}{c}=2$.

3. A variable plane which is at a constant distance 3p from the origin "O" cuts the axes at A,B,C. Show that the locus of the centroid of the triangle ABC is $x^{-2}+y^{-2}+z^{-2}=p^{-2}$

4. A triangle, the lengths of whose sides are a,b and c is placed so that the middle points of the sides are on the axes. Show that the equation to the plane is $\frac{x}{\alpha}+\frac{y}{\beta}+\frac{z}{\gamma}=1$ where $8\alpha^2=b^2+c^2-a^2, 8\beta^2=c^2+a^2-b^2, 8\gamma^2=a^2+b^2-c^2$ and find the coordinates of its vertices.

5. A variable plane passes through a fixed point (α,β,γ) and meets the coordinate axes at P, Q and R. Show that the locus of the point common to the planes through P,Q,R and parallel to the coordinate planes is $\frac{\alpha}{x}+\frac{\beta}{y}+\frac{\gamma}{z}=1$.

6. A variable plane at a constant distance p from the origin meets the axes at A,B,C. Show that the locus of the centroid of the tetrahedron OABC is $x^{-2}+y^{-2}+z^{-2}=16p^{-2}$

* Sphere (Each question carries 5 marks)

1. Obtain the equation of the sphere passing through four non coplanar points (x_i, y_i, z_i) $(i=1,2,3,4)$

2. Prove that the circles $x^2+y^2+z^2-2x+3y+4z-5=0$, $5y+6z+1=0$ and $x^2+y^2+z^2-3x-4y+5z-6=0$, $x+2y-7z=0$ lie on the same sphere, whose equation is $x^2+y^2+z^2-2x-2y-2z-6=0$

3. Find the equation of the sphere which passes through the circle $x^2+y^2+z^2-2x-3y+4z+8=0=x^2+y^2+z^2+4x+5y-6z+2$ and having its centre on the plane $4x-5y-z=3$

4. Show that the equation of the sphere which passes through the circle $z=0$, $x^2+y^2=a^2$ and through the point (α,β,γ) is $\gamma(x^2+y^2+z^2-a^2)=z(\alpha^2+\beta^2+\gamma^2-a^2)$.

5. Obtain the greatest and least distance of the point $(1,-1,2)$ from the sphere $x^2+y^2+z^2-4x+6y-8z=71$

6. Define great circle of a sphere. Find the equation of the sphere having the circle $x^2+y^2+z^2-2x+4y-6z+7=0$, $2x-y+2z=5$ as a great circle

7. Show that the sphere $x^2+y^2+z^2+2u_1x+2v_1y+2w_1z+d_1=0$ cuts the sphere $x^2+y^2+z^2+2u_2x+2v_2y+2w_2z+d_2=0$ in a great circle, if $2(u_1u_2+v_1v_2+w_1w_2)=2r^2+d_1+d_2$, where r is the radius of the second sphere

8. A variable plane passes through a fixed point. Show that the locus of the foot of the perpendicular from the origin to the plane is a sphere.

9. A variable sphere passes through the points $(0,0,\pm c)$ and cuts the straight lines $y=x\tan\alpha$, $z=c$; $y=-x\tan\alpha$, $z=-c$ in the points P and P'. If $PP'=2a$, a constant, then show that the centre of the sphere lies on the circle $z=0$, $x^2+y^2=(a^2-c^2)\operatorname{cosec}^2 2\alpha$

10. OA, OB and OC are three mutually perpendicular straight lines through the origin and their direction cosines are l_1, m_1, n_1; l_2, m_2, n_2, l_3, m_3, n_3. If $OA=a$, $OB=b$, $OC=c$, then prove that the equation of the sphere OABC is
$$x^2+y^2+z^2-x(al_1+bl_2+cl_3)-y(am_1+bm_2+cm_3)-z(an_1+bn_2+cn_3)=0$$

11. If a sphere touches the planes $2x+3y-6z+14=0$ and $2x+3y-6z+42=0$ and if its centre lies on the straight line $2x+z=0, y=0$, find the equation of the sphere.

12. Find the equations of the spheres touching the three co-ordinate planes.

13. If any tangent plane to the sphere $x^2+y^2+z^2=r^2$ makes intercepts a, b and c on the co-ordinate axes, then prove that $a^{-2}+b^{-2}+c^{-2}=r^{-2}$.

14. Find the sphere of the smallest radius that touches the straight lines $\frac{x-2}{1}=\frac{y+1}{-2}=\frac{z-6}{1}$ and $\frac{x+3}{7}=\frac{y+3}{-6}=\frac{z+3}{1}$

15. Show that there is no tangent plane to the sphere $x^2+y^2+z^2-4x+2y-4z+4=0$ that can be passed through the straight line $\frac{x+6}{2}=\frac{y+3}{1}=\frac{z+1}{1}$

16. Prove that the centres of spheres which touch the straight lines $y=mx, z=c$ and $y=-mx, z=-c$ lie on the surface $mxy+cz(1+m^2)=0$

Quadric Surfaces (Each question carries 5 marks)

(A) Cone

1. Show that the general equation of a cone with vertex at origin is $ax^2+by^2+cz^2+2fyz+2gzx+2hxy=0$ where $\Delta=\begin{vmatrix} a & h & g \\ h & b & f \\ g & f & c \end{vmatrix} \neq 0$.

2. Obtain the condition that the general second degree equation in x, y, z: $ax^2+by^2+cz^2+2fyz+2gzx+2hxy+2ux+2vy+2wz+d=0$, represents a cone.

3. Find the equation of the cone whose vertex is the point $(1,1,1)$ and base is the curve $x^2+y^2=16$, $z=0$.

4. Define a right circular cone. Show that the equation of the right circular cone whose vertex is the point $(0,0,3)$ and whose guiding curve is $x^2+y^2=4$, $z=0$ is $9(x^2+y^2)=4(z-3)^2$.

5. A cone has for its guiding curve the circle $x^2+y^2+2ax+2by=0$, $z=0$ and passes through a fixed point $(0,0,c)$. If the section of the cone by the plane $x=0$ be a rectangular hyperbola, then prove that the vertex lies on the fixed circle $x^2+y^2+z^2+2ax+2by=0$, $2ax+2by+cz=0$.

6. Show that the angle between the straight lines $x+y+z=0$, $ayz+bzx+cxy=0$ is 90° if $a+b+c=0$ and 60° if $a^{-1}+b^{-1}+c^{-1}=0$ $(abc \neq 0)$.

7. Show that the equation of the cone which passes through the co-ordinate axes and the straight lines $\frac{x}{1}=\frac{y}{-2}=\frac{z}{3}$ and $\frac{x}{3}=\frac{y}{2}=\frac{z}{-1}$ is $3yz+10zx+6xy=0$.

8. A variable plane passing through the x-axis and a variable plane passing through the y-axis are inclined at a constant angle α. Prove that their line of intersection generates the cone $z^2(x^2+y^2+z^2)=x^2y^2\tan^2\alpha$.

9. Find the equation of the cone obtained by revolving the straight line $x-2y-1=0$ in the plane $z=0$.

10. Show that the locus of points from which three mutually perpendicular straight lines can be drawn to intersect the conic $ax^2+by^2=1$, $z=0$ is $ax^2+by^2+(a+b)z^2=1$.

11. If $2x=y=2z$ represents one line of the set of three mutually perpendicular generators of the cone $11yz+6zx-14xy=0$, then find the equations of the other two.

12. Prove that the conditions that the lines of section of the plane $lx+my+nz=0$ and the cones $ax^2+by^2+cz^2=0$, $fyz+gzx+hxy=0$ may be coincident, are

$$\frac{bn^2+cm^2}{fmn} = \frac{cl^2+an^2}{gnl} = \frac{am^2+bl^2}{hlm}$$

13. The section of the cone whose guiding curve is the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$, $z=0$ by the plane $x=0$ is a rectangular hyperbola. Show that the locus of the vertex is the surface $\frac{x^2}{a^2}+\frac{y^2+z^2}{b^2}=1$

14. Show that the straight lines in which the plane $ux+vy+wz=0$ cuts the cone $ax^2+by^2+cz^2=0$ are perpendicular, if $(b+c)u^2+(c+a)v^2+(a+b)w^2=0$ and parallel, if $bc\,u^2+ca\,v^2+ab\,w^2=0$.

15. Obtain the condition of tangency of the plane $ux+vy+wz=0$ to the cone $ax^2+by^2+cz^2+2fyz+2gzx+2hxy=0$

16. Prove that the lines of intersection of pairs of tangent planes to the cone $ax^2+by^2+cz^2=0$ which touch along perpendicular generators lie on the cone $a^2(b+c)x^2+b^2(c+a)y^2+c^2(a+b)z^2=0$.

17. Show that the equation of the reciprocal cone of the cone whose vertex is $(0,0,c)$ and base is $x^2+y^2-2ax=0$, $z=0$ is $a^2y^2-c^2(z-c)^2+2cax(z-c)=0$.

18. Prove that the planes which cut the cone $ax^2+by^2+cz^2=0$ in perpendicular straight lines, touch the cone

$$\frac{x^2}{b+c}+\frac{y^2}{c+a}+\frac{z^2}{a+b}=0$$

19. Show that the plane $z=a$ meets any enveloping cone of the sphere $x^2+y^2+z^2=a^2$ in a conic which has a focus at the point $(0,0,a)$.

20.

(B) Cylinder (Each question carries 5 marks)

1. Find the equation of the cylinder whose generators are parallel to the straight line $\frac{x}{2} = \frac{y}{3} = \frac{z}{5}$ and which passes through the conic $z=0,\ 3x^2+7y^2=12$.

2. Obtain the equation of the cylinder whose generators intersect the ellipse $9x^2+3y^2=1,\ z=0$ and are parallel to the straight line with direction ratios $1, -1, 1$.

3. Define right circular cylinder. Find the equation of the right circular cylinder which passes through the point $(3,-1,1)$ and has the straight line
$\frac{x-1}{2} = \frac{y+3}{-1} = \frac{z-2}{1}$ as its axis.

4. Find the equation of the right circular cylinder whose axis is $\frac{x}{1} = \frac{y}{-2} = \frac{z}{2}$ and radius equal to $\sqrt{6}$

5 Find the equation of the right circular cylinder whose axis is the straight line which passes through the point $(1,3,4)$ and has $1, -2, 3$ as its direction ratios and radius equal to 3

6. Show that the equation of the enveloping cylinder of the sphere $x^2+y^2+z^2-2x+4y-1=0$ having generators parallel to the straight line $x=y=z$ is
$x^2+y^2+z^2-yz-zx-xy-4x+5y-z-2=0$

(C) Conicoids (Each question carries 5 marks)

1. Find the values of p for which the plane $x+y+z=p$ is a tangent plane to the ellipsoid $x^2+2y^2+3z^2=66$. For each one of these values of p, find the co-ordinates of the point of contact

2. Show that there are two tangent planes to the quadric $\frac{x^2}{42} - \frac{y^2}{18} + \frac{z^2}{3} = 1$ which pass through the line $\frac{x-8}{-7} = \frac{y+4}{5} = \frac{z-1}{1}$,

3. Find the points of contact of the tangent planes to the conicoid $2x^2-25y^2+2z^2=1$ which pass through the line joining the points $(-12,1,12)$ and $(13,-1,-13)$.

4. If $2r$ be the distance between the two tangent planes to the ellipsoid $\frac{x^2}{a^2}+\frac{y^2}{b^2}+\frac{z^2}{c^2}=1$ which are parallel to the plane $lx+my+nz=0$, then show that
$(a^2-r^2)l^2+(b^2-r^2)m^2+(c^2-r^2)n^2=0$

5. Find the locus of a luminous point, if the ellipsoid $\frac{x^2}{a^2}+\frac{y^2}{b^2}+\frac{z^2}{c^2}=1$ casts a circular shadow on the plane $z=0$.

6. Show that the locus of a point from which the ~~three~~ mutually perpendicular tangent planes can be drawn to touch the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$, $z=0$ is the sphere $x^2+y^2+z^2=a^2+b^2$

7. Prove that the locus of the feet of the perpendiculars drawn from the centre of the ellipsoid $\frac{x^2}{a^2}+\frac{y^2}{b^2}+\frac{z^2}{c^2}=1$ upon the tangent planes is

$$a^2x^2+b^2y^2+c^2z^2=(x^2+y^2+z^2)^2$$

8 Show that, in general, six normals can be drawn from a point to the conicoid $ax^2+by^2+cz^2=1$.

9 Find the equation of the cone through the six normals drawn from the point (f,g,h) to the ellipsoid $\frac{x^2}{a^2}+\frac{y^2}{b^2}+\frac{z^2}{c^2}=1$

10. Show that the points $(12,-18,8)$ and $(-6,18,-10)$ are at the feet of the normals to the ellipsoid which lie on the plane $x+y+z=2$. $x^2+2y^2+3z^2=984$

11. Show that, in general, three normals can be drawn from a given point to the paraboloid $x^2+y^2=2az$, but if the point lies on the surface $27a(x^2+y^2)$ $+8(a-z)^3=0$, then two of them coincide.

12 If the normal at the point P to the ellipsoid $\frac{x^2}{a^2}+\frac{y^2}{b^2}+\frac{z^2}{c^2}=1$ meets the principal planes in G_1, G_2, G_3, then show that $PG_1 : PG_2 : PG_3 = a^2 : b^2 : c^2$

13. If the length of the normal chord through the point P of the ellipsoid $\frac{x^2}{a^2}+\frac{y^2}{b^2}+\frac{z^2}{c^2}=1$ be equal to $4PG_3$ where G_3 is the point in which the normal meets the plane $z=0$, then show that P lies on the cone

$$\frac{x^2}{a^6}(2c^2-a^2)+\frac{y^2}{b^6}(2c^2-b^2)+\frac{z^2}{c^4}=0$$

14. If the feet of the six normals from the point (α,β,γ) to the ellipsoid $\frac{x^2}{a^2}+\frac{y^2}{b^2}+\frac{z^2}{c^2}=1$ be (x_i,y_i,z_i) $(i=1,2,3,\ldots,6)$ then prove that $a^2\alpha\sum_{i=1}^{6}x_i^{-1}+b^2\beta\sum_{i=1}^{6}y_i^{-1}+c^2\gamma\sum_{i=1}^{6}z_i^{-1}=0$.

15 Show that the feet of the normals from the point (α,β,γ) to the ellipsoid $\frac{x^2}{a^2}+\frac{y^2}{b^2}+\frac{z^2}{c^2}=1$ lie on the intersection of the ellipsoid and the cone

$$\frac{\alpha a^2(b^2-c^2)}{x}+\frac{\beta b^2(c^2-a^2)}{y}+\frac{\gamma c^2(a^2-b^2)}{z}=0$$